Struve Geodetic Arc
2006 International Conference

»The Struve Arc and Extensions in Space and Time»
Haparanda and Pajala, Sweden, 13-15 August 2006

Edited by
Hans-Fredrik Wennström

Gävle 2006
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Baselines and meridians–some short notes

Ian Brook
Geodesist, formerly National Land Survey of Sweden
ian.brook@surcon.se

During the mid 1950s and late 1960s I was involved in measurements related to three major international co-ordinate reference systems: the Tromsø-Catania satellite triangulation baseline, the Swedish-Finnish first-order triangulation in Tomedalen along the line of Struve’s meridian arc and check measurements of the Ugandan primary triangulation in Uganda, the adjustment of which was based on the arc of the 30th meridian.

The measurement of the Swedish-Norwegian section of the Tromsø - Catania satellite baseline

During the 1960s the American National Oceanic and Atmospheric Administration (NOAA) established a world-wide network of satellite triangulation stations by measuring the positions of passive satellites against a fixed star background. To determine the scale of the network a number of terrestrial baselines were required. The Geographical Survey Office of Sweden – since the mid-1970s part of the National Land Survey of Sweden – was requested to assist NOAA by measuring the section of the Tromsø (Station 6) - Catania (Station 16) baseline through Sweden and Norway and the cross-water connection to Denmark. In addition, sides in Belgium and Switzerland were measured with the laser Geodimeter. I was responsible for carrying out the field part of the programme.
In the 1967 field season the 56 sides which comprise the Norwegian and Swedish section of the baseline were measured. Accuracy requirements were, naturally, high and NOAA made available high precision EDM equipment, namely two third generation, model MRA4 Tellurometers and a modified model 4D Geodimeter in which the standard light source had been replaced by a 2 milliwatt helium-neon gas laser. This was the first time a laser Geodimeter had been used in Europe for this type of measurement.

The route of the traverse through Sweden is shown on the sketch map of the Swedish first-order triangulation. As can be seen, the distance measurements have been carried out as a single continuous traverse which follows the backbone chain of the first-order network. The line of the traverse was, to a certain extent, determined by the need to minimise expensive tower building and to make use of existing towers. Between the final Swedish station in the north and Tromsø in Norway, the traverse follows a part of the Norwegian first-order trilateration net-work. In the middle and southern parts of the traverse which runs through forested areas, special wooden, distance measuring towers were, however, necessary. From station 225 northwards the traverse skirts the eastern edge of the mountainous areas which make up the western part of Sweden at these latitudes. The average height of the stations increased northwards to a maximum elevation of over 1,600 metres in the middle of the Norwegian mountain section, thereafter decreasing rapidly to 100 m over sea level at the terminal camera station in Tromsø.

As transportation presented considerable problems in the mountain areas, two helicopters were used for transporting observers and equipment. For the remainder of the traverse, tough back-packing was necessary. Progress in this part of the traverse was slower due largely to adverse weather conditions; and the final connection to Denmark was completed in the 25th October. This connection was made in co-operation with the Danish Geodetic Institute. The total length of the traverse was 1,844,558 metres.

Observations
Measuring routines were based on experience gained from a long series of test measurements carried out in a special test network designed for testing and evaluating EDM equipment. To maintain a high level of accuracy, all sides in the traverse were measured using the Tellurometers and check measurements were made at regular intervals using the laser Geodimeter. The observation programme began at the beginning of July at station 265 and during the first month the rate of progress northwards was approximately 150 km per day i.e. four Tellurometer-measured distances. Progress was considerably slower in the Norwegian
mountains where very poor weather often grounded the helicopters. The connection to the Nordlyse camera station in Tromsø was measured at the end of July. It had been planned to measure all Tellurometer sides three times but the continued poor weather in Norway made it impossible to measure the sides between station 550 and 5481 more than once. These distances had, however, been measured previously by the Geographical Survey of Norway using MRA 5 Tellurometers. During the period 5th July - 31st August, 41 Tellurometer distances were measured. As all sides, except the five most northerly, were measured three times, this represented a total distance measured with the Tellurometer of 4 528 791 metres or 87 000 metres per working day. In all, 50 distances were measured using the Tellurometers.

Seven of the distances were checked using the laser Geodimeter and six of them had previously been measured using a Model 2 Geodimeter – by Erik Bergstrøm, the father of the Geodimeter. In the southern part of the traverse five sides, which had previously been measured to first-order standards using standard Model 4D Geodimeters were incorporated in the traverse. The connection to Denmark was measured by Tellurometer and, later, a check measurement was made with the laser Geodimeter. This distance is 47 000 metres. Additional check Geodimeter measurements were made during the 1968 field season.

**Instrument calibration**

All equipment – EDM instruments and the meteorological equipment used for determining pressure and wet and dry bulb temperatures - was carefully calibrated before, during and after the field season. Zero error corrections for both the Tellurometers and Geodimeters were determined before the beginning of the field work and, in addition a further zero error determination was carried out before the connection to Denmark was made. Crystal frequencies were checked at regular intervals.

**The results**

Before the data was delivered to NOAA a check computation was carried out. In addition to the new data from the traverse, all available first-order angle observations and Laplace observations were included in the adjustment.

The standard error a priori for an observed direction was 1° and for distances – MRA4 Tellurometer and Geodimeter – 101 s.10^-6 m. The distance between Tromsø and the crossing to Denmark at Högkull is approximately 1 524 km.
A first-order chain in Tornedalen

In 1969 a joint team from the Finnish Geodetic Institute and the Geographical Survey of Sweden measured a new primary chain of triangulation along Tornedalen to connect the Finnish and Swedish triangulation networks in this area. I was in charge of the Swedish party and carried out the laser Geodimeter measurements.

The chain generally followed the same line as Struve's meridian arc. Although no efforts were made to locate and connect to Struve's points due to weather and financial constraints, observations were made on Pullinki in Sweden and Gunastunturi in Finland.

The programme also had a scientific component inasmuch as several distances in the chain were measured simultaneously with both laser Geodimeter and MRA 101 and MRA3 Tellurometers for comparative purposes. Microwave measurements are generally more sensitive to water vapour pressure and computation of representative values for the refractive index are consequently more difficult than for Geodimeters. The Finnish party carried out experiments as the basis for research on temperature gradients when making the meteorological observations.

Twenty-five sides were measured with both instrument types. The sides were measured twice on the same day with the laser Geodimeter, with the Tellurometer, two measurements were made on two separate days. The maximum side length measured was 57 000 m. Computations of the distance measurements showed that the degree of agreement between the two types of instruments was, with two exceptions, good: the average side length was 31 000 metres and the average difference, Tellurometer minus Geodimeter was 50mm. There appeared, however, to be a tendency for the Tellurometer distances to be systematically shorter than the Geodimeter distances. I am not aware of the results of the research that was later done on refractive index at the Geodetic Institute and how the results affected the Tellurometer distances. All measurements were made in daylight, many under bright, hot, sunny conditions. The fifty measurements were made on fourteen observing days which is an interesting comparison with Struve's and other campaigns in Tornedalen!

Both parties were very much aware of the significance of Tornedalen in geodetic history and in whose footsteps we were treading. My experience from Africa, Europe and other countries is that if you ask two surveyors to establish a point on the top of a hill and they will invariably place their points close to the same spot.
The 30th Arc

In 1954 I joined the Lands and Surveys Department in Uganda as a staff surveyor. By that time the very difficult connection that finally completed the 30th Arc from the Cape to Cairo had been completed between the southern Sudan and northern Uganda by the American Army Map Service. The geocetic spine of Africa was, thereby complete. In 1951 an adjustment of the Arc between the then Southern Rhodesia and Uganda had been completed and the New (1950) Arc Datum established. A computation of the Ugandan primary network was referred to this datum.

Although I was not involved in these Arc measurements, some years later I was seconded to the Directorate of Overseas Surveys which, at that time, was establishing a new primary triangulation network and carrying out topographic mapping in Uganda, to assist in carrying out measurements to attempt to locate scale errors in the triangulation. Astronomical observations were also carried out. The distance measurements were carried out with Tellurometer equipment. The introduction of the Tellurometer in 1957 was a major step forward in geodetic technology and techniques. With this microwave instrument it was possible to measure distances up to 50-60 km with good accuracy even in poor visibility. In addition, the instrument was very portable, easy to operate and had a built-in communication system, which was of immense value in African conditions. The Geodimeter system - an electro-optical system - had been in use for some years when the Tellurometer was launched and had, for example, been used in Sweden for scale determinations of long sides in the Swedish first-order network. The Geodimeter models that were available in the 1950s were, however, heavy and cumbersome, not easy to operate and could only be used for measuring long distances at night with good visibility. The Geodimeter at that time was inherently a more accurate instrument than the Tellurometer but was hardly field equipment.

I carried out a number of check measurements for DOS. Looking back it is clear that knowledge of the operation of Tellurometers at that time was very limited: of cyclic errors, swing phenomena, the various velocity questions and, above all, the critical importance of the determination of temperatures. The results could have been much better.

A re-computation of the Uganda triangulation was later done in 1960 to fit with the coordinates obtained from the re-adjustment of 30th Arc. In 1964 the results of the last East African triangulation were published and designated New (1960) Arc.

And now the next major technological step forward, the introduction of GPS, has been taken. GPS has certainly made life easier for the geodetic surveyor - fewer hills to climb with
heavy loads, fewer towers to build, no need for chains of triangulation, easier connections between continents and less dependence on the elements. But also less charm, less physical and mental satisfaction in particular for those who have been privileged to stand on a hill in Africa waiting for night to fall and signal lamps beginning to show around the horizon so angle observations can begin or who have measured distances on clear Nordic summer nights.
Struve Geodetic Arc in Lithuania

Arunas Buga, Petras Petroskevicius
Institute of Geodesy, VGTU, Sauletekio al. 11, LT-10223 Vilnius, Lithuania
buga@ap.vtu.lt; petras.petroskevicius@ap.vtu.lt

Vitalija Juceviciute
National Land Service, Gedimino pr.11, LT-01103 Vilnius, Lithuania
vitalijaj@zum.lt

Introduction

The Struve geodetic arc, laying almost on the boarder of Western and Eastern Europe and passing from the Arctic Ocean to the Black Sea, was measured in the first half of the 19th century. It is a chain of triangulation triangles, running almost through meridian of 26° East of Greenwich from Hammerfest in the North of Norway to the Ismail close to the Black Sea.

Length of arc is 2,822 km. Triangulation chain is composed of 258 triangles with special marks at the vertices. Mean side length of triangle is 27 km, but there are sides with length over 50 km. It passes through modern 10 countries: Norway, Sweden, Finland, Russian Federation, Estonia, Latvia, Lithuania, Belarus, Moldova and Ukraine. The Struve arc in total consists of 265 points. The largest number of points – 83 is in Finland, while in Lithuania – 18.

Chain of triangulation was surveyed in 1816–1855.

F. G. W. Struve professor of mathematics and astronomy at the Dorpat (Tartu, Estonia) University was supervising the establishment of geodetic arc. Therefore geodetic arc is named after him.
F. G. W. Struve – supervisor of Geodetic Arc observations

F. G. W. Struve, a famous astronomer and geodesist, was born in 1793 in Altona town, close to Hamburg, in the family of gymnasium’s principal. He did not finish studies at the gymnasium and entered the Dorpat University in 1808. Struve has graduated from philology faculty after two and a half years and chose a new speciality i.e. astronomy. He was granted Master and later Doctor of Philosophy degree for the defended dissertation on the Dorpat observatory geographic location in 1813. He was appointed professor of mathematics and astronomy as well as associate of the Dorpat University Observatory. He was one of the first astronomers to measure the stellar parallax and he has discovered a number of double stars. F. G. W. Struve was put in charge of geodetic observations in Livonia (area covering present territories of Estonia and Latvia) in 1816.

F. G. W. Struve took active care in preparations for building Pulkovo observatory and became its first director in 1839. Struve took care of purchasing astronomic instruments, prepared programs of scientific works, continued research of stars which had been earlier started, determined some astronomic constants and dealt with other astronomic problems. He prepared and published catalogues of double stars coordinates.

Despite full employment in astronomic activities he paid much attention to the geodetic work and research. Struve described his geodetic observations and their results in his two-volumes publication „Geodetic Arc“. He was in charge of some chronometric expeditions for determination of longitude difference between Pulkovo and Greenwich. Struve has published over the 150 scientific publications, most of them in the field of geodesy and practical astronomy.

Struve became a correspondent member of St. Petersburg Academy of Sciences in 1822 and its member in 1832. He was among founders of the Russian Geographic Society and head of a section of mathematical cartography. Struve was lecturing teaching geodesy and astronomy to the officers of military topography for 25 years since 1824.

The purpose for measuring the Struve Geodetic Arc was to determine the form and size of the Earth ellipsoid. The determination of Earth’s form and size was one of the most important problems ever since Aristotle’s time (4th century BC).
Geodetic observations of Struve Arc in Lithuania

Measurements of Lithuanian part of Struve geodetic arc were started in 1816 by surveying triangulation network for Vilnius province, conducted mainly for mapping. The Eastern boarder of the province reached Breslauja, Ashmiany, Lida and Grodno, the Southwestern boarder went along Nemunas river and reached Palanga in the West. The Northern boarder was very similar to the present national boarder between Lithuania and Latvia. This triangulation network was designed and measurements were organized by Tsar army officer of Estonian descent Carl Tenner. Network observations were completed in 1821. He started them on his own initiative, but later received approval and guidance from the government. Arc observations were started along the meridian of Vilnius observatory and continued northwards and southwards for almost 35 years. Tenner supervised establishment of triangulation chain from Jekabpils (Latvia) to the mouth of the river Danube. The Vilnius University graduate J. Chadzka (1800-1881) assisted Tenner in these observations. Part of the arc, from the point BRISTEN (56°35') in Latvia to the point BELIN (52°03') in Belarus, completed by Tenner in 1827, is called ‘Lithuanian Arc’.

Table 1. Struve arc points in Lithuanian territory

<table>
<thead>
<tr>
<th>Point No.</th>
<th>Original point name</th>
<th>Modern point name</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Locality</th>
</tr>
</thead>
<tbody>
<tr>
<td>143</td>
<td>Chanuschiski</td>
<td>-</td>
<td>56°08'46&quot;</td>
<td>25°31'48&quot;</td>
<td>Rokiskis district Omskis village</td>
</tr>
<tr>
<td>144</td>
<td>Pondelē</td>
<td>Pandeis</td>
<td>56 01 43</td>
<td>25 13 26</td>
<td>Rokiskis district Pandeles town</td>
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<tr>
<td>145</td>
<td>Jokšiški</td>
<td>Jokšia</td>
<td>55 55 55</td>
<td>25 08 47</td>
<td>Kupiskis district Jokšiai village</td>
</tr>
<tr>
<td>146</td>
<td>Girešiai</td>
<td>Gireisiai</td>
<td>55 54 09</td>
<td>25 26 12</td>
<td>Rokiskis district Gireisiai village</td>
</tr>
<tr>
<td>147</td>
<td>Martinchekas</td>
<td>Martincichai</td>
<td>55 43 28</td>
<td>25 37 25</td>
<td>Utena district Martinciai village</td>
</tr>
<tr>
<td>148</td>
<td>Kindertų</td>
<td>Kinderiai</td>
<td>55 46 27</td>
<td>25 06 30</td>
<td>Kupiskis district Kinderiai village</td>
</tr>
<tr>
<td>149</td>
<td>Lipškis</td>
<td>Kuzniskis</td>
<td>55 32 22</td>
<td>25 23 46</td>
<td>Anykščiai district Austra village</td>
</tr>
<tr>
<td>150</td>
<td>Stverėnių</td>
<td>Storiai</td>
<td>55 29 19</td>
<td>25 09 00</td>
<td>Anykščiai district Vilniškiškės village</td>
</tr>
<tr>
<td>151</td>
<td>Tšišiviški</td>
<td>Civiliški</td>
<td>55 19 06</td>
<td>25 28 13</td>
<td>Moletai district Civiliški village</td>
</tr>
<tr>
<td>152</td>
<td>Bulšiki</td>
<td>Petrüškių</td>
<td>55 16 58</td>
<td>25 09 13</td>
<td>Moletai district Petrųškių village</td>
</tr>
<tr>
<td>153</td>
<td>Ambroschiski</td>
<td>Ambraziskiai</td>
<td>55 08 24</td>
<td>25 18 24</td>
<td>Moletai district Ambraziskiai village</td>
</tr>
<tr>
<td>154</td>
<td>Kongiedi</td>
<td>Kungiedai</td>
<td>55 06 31</td>
<td>24 46 54</td>
<td>Sirvintos district Jaskužyai village</td>
</tr>
<tr>
<td>155</td>
<td>Meseikai</td>
<td>Meskonys</td>
<td>54 55 51</td>
<td>25 19 00</td>
<td>Vilnius district</td>
</tr>
</tbody>
</table>
Carl Tenner — famous geodesist of Russia, was born close to Narva in a poor family. While initially having only elementary education, he gained knowledge of geodesy and cartography during his army service and studying geodetic literature by himself. He was involved in St. Petersburg and Gulf of Finland Southern shore triangulation in 1812. After establishing triangulation in Vilnius province, he continued the same work in provinces of Courland, Grodno, Minsk, Volhynia, Podol, Kiev and Bessarabia. He connected the network that he had established himself with the triangulation networks of Eastern Prussia and Austria. Together with his assistants, he measured almost half of Struve geodetic arc. Many baselines were measured thanks to Tenner’s assistance. Part of the astronomical observations in the Southern part of the arc was done by Tenner as well as angular measurements and most computations of geodetic observations. Tenner determined over 4500 triangulation points, more than 700 of them were the first order points. After the trigonometric levelling, he determined heights of triangulation points and the level difference between Baltic and Black Seas. He was in charge of topographic survey along with triangulation activities. Having completed triangulation network in Poland, he finished geodetic activities in Tsar’s army in the rank of general in 1858. Tenner’s observations were of highest quality and were greatly valued by Struve and the famous German astronomer, director of Königsberg observatory Friedrich Wilhelm Bessel (1784–1846). K. Tenner was elected a correspondent member of St. Petersburg Academy of Sciences.
Fig 1. The part of Struve arc in the Lithuanian territory (Struve F. G. W. „Arc du méridien de 25° 20’ entre le Danube et la Mer Glaciale mesure depuis 1816 jusqu’en 1855“, St Petersburg, 1860)

Struve started the arc observations in Livonia by his own. He used to ask advices on geodetic observations from F. W. Bessel and C. F. Gauss. Struve engaged the Russian government in the geodetic arc of 3.5° observations between island of Gogland in Gulf of Finland and Jekabpils in the South. This arc was observed from 1822 to 1831. Astronomic observations in points JEKABPILS and HOGLAND were assisted by professor G. Paucker from Mitau (Jelgava, Latvia). The most complicated observations were those in the Gulf of Finland,
where the longest triangle sides were over 80 km. This chain connected many previous triangulation networks of Livonia, established by Struve in 1816–1819. In 1829, Tenner connected the arc measurements made by Struve and himself. Struve aimed at extending the length of the arc and succeeded to get governmental support and resources for continuation of his and Tenner’s activities. Struve started observations in Finland in 1830. The observations under Struve’s supervision continued there until 1851. The observations in Finland were complicated due to forests and the swamplands.

The St. Petersburg Academy of Science asked for support from institutions of Sweden and Norway in order to extend the triangulation chain to the Northern shores of Norway. The address was answered by the director of Stockholm Royal Observatory N. H. Selander and the director of Christiania (Oslo) Observatory C. Hansteen. With assistance of astronomers Lindhagen and Vagner they completed the planned chain in 1852.

In 1855 the geodetic arc observations were completed. Struve suggested to extend triangulation chain southwards, but due to the political unrest in the region this was not achieved.

The point of the triangulation chain were monumented by the laid fieldstones with limewash in the dug pit. In the top stone the hole was drilled. The hole was filled with lead. Its centre was marked by a cross. On the ground, the centre was marked by a pole with a driven nail. In the Northern part of the Struve geodetic arc many triangulation points were marked on the solid rock by drilling a hole.
Wooden pyramids of 5–11 m height were built on the hills, while in the woods the geodetic towers were built to lift the instruments. Construction of Tenner’s triangulation towers used to be simple. Their mean height was 19 m. Only the highest ones reached 30–40 m.

Tenner measured most of the triangulation triangles in Lithuania by using two repeating circles, one by Baumann which read to 4" by vernier, and the other by Throughton reading by vernier to 10". Telescopes magnification was to 25–27 times. Reichenbach’s repeating circle theodolite belonging to the Vilnius observatory was used as well.
G. Littel's simple and astronomical theodolites, as well as universal instrument, were used for the Southern part of the arc observations. Struve used theodolites and a universal instrument by Reichenbach of Munich for angular measurements. Tenner used a method of repetition from the very beginning. Every angle he measured separately up to 20-50 times. Later on he used new method suggested by Struve. The allowed closing errors of triangles were up to 3".
The baselines of triangulation chain were measured by the baseline instruments. There were four iron bars in Tenner's instruments. Length of every bar was 14 feet (about 4.27 m). For the baseline length measurement, bars used to be laid in a line on the balks, while these on the wooden bucks. Gaps between bars used to be measured by a ruler with vernier, by which, using a magnifying glass, 0.001 inch (0.025 mm) accuracy was achieved. Temperature of the bars was at first measured by bimetallic thermometer, and later by a mercury thermometer. Slope angles used to be measured by clinometer with a level. Accuracy of 0.1° was reached. Relative errors of the measured baselines were around 1/300 000. The longest baseline in the Struve arc chain was 11.8 km length. It was measured in 1820 between Pandelys town and Jokšiai village. Struve used a baseline instruments that he had himself improved.

There was a problem in the arc triangulation chain related to the different units of length: Russian-English sajen (7 feet), used by Tenner, and Russian-French loise (6 feet), used by Struve.

At the Struve geodetic arc astronomic points by means of celestial bodies observations the astronomic latitude and astronomic azimuth of direction towards the neighbouring point were determined. Temporary astronomic observatories with stone poles for the instruments used to be frequently built for the astronomic observations.

The passage instrument from Vilnius observatory by the famous British mechanic J. Ramsden and astronomic clock by J. Shelton along with other instruments were used at the astronomic points BRISTIEN, NEMEZIS and BELLIN.

It should be stressed that astronomers of the Vilnius observatory actively participated in the above-mentioned astronomic observations. Director of Vilnius observatory Jonas Sniadeckis (1756–1830) and Petras Slavinskis determined the astronomic latitude difference between the stations at MESKONYS and that of Vilnius observatory. They observed at the Vilnius observatory while K. Tenner with his assistants measured at the point MESKONYS. The local time at the points was determined by the astronomic observations. Difference in the local time as well as that of longitude was determined by observing flashes of burned gunpowder on the hill seen from the both points. 86 flashes were observed during 12 days. By these observations the astronomic longitude of the point MESKONYS in relation to Paris was determined as $\lambda = 22^\circ 58' 02,55"$. At the same year the astronomic azimuth of direction MESKONYS NEMEZIS and latitude at the point MESKONYS were determined. The data was used for the initial orientation of the triangulation networks at provinces of Vilnius, Courland and Grodno.
P. Slavinskis assisted by J. Chodzka determined astronomic latitude of triangulation point LIDINTAIČIAL (around 9 km northwards from Telsiai) in 1824. George Reichenbach vertical circle, just obtained by the observatory, was used in order to determine the latitude. Totally 700 zenith distances to the stars have been measured.

In 1826, director of observatory P. Slavinskis with astronomer M. Hlusnevicius (1797-1862) and M. Zilinskis determined latitude of the astronomic point BRISTEN. Observations lasted one month. Observations were made a passage instrument. Method suggested by Bessel was used, according to which 36 stars on first vertical were observed. Local time and clock run were determined by the passage instrument. This method of latitude determination was used in other astronomic points of Lithuanian arc as well.

A temporary astronomic observatory was established in Nemezis astronomic point, 6 km southeast from Vilnius observatory and the first observations were made in 1826. Nemezis astronomic latitude was determined by M. Hlusnevicius and J. Chodzka in 1827. The passage instrument was used. Observations lasted from August 2nd to October 13th. During 48 nights, 86 stars were observed for the latitude determination.

In 1827, Tenner, J. Chozka and M. Zilinskas conducted the astronomic observations in BELIN point.

Longitude of NEMEZIS point was computed in 1828 through the triangulation network from Dorpat observatory.

Azimuth of the side NEMEZIS-MESKONYS was determined using the passage instrument in 1832. Repeated observations of the three brightest stars of the Little Bear were made by astronomer of Vilnius observatory M. Hlusnevicius. Struve computed the azimuth. He and Tenner highly valued the observations made by M. Hlusnevicius. In response to the application from Struve and Tenner M. Hlusnevicius was awarded with St. Stanislav IV rank order and a diamond ring by the Russian Tsar.

Longitude of Dorpat was re-determined in 1854. Struve was in charge of the chronometric expedition between Dorpat and Pulkovo. 31 chronometers were transported 10 times.

Latitude of Nemezis was redetermined by the director of Vilnius observatory Jegoras Sableris (1810–1865) in 1855. The G. Reichenbach’s vertical circle from Vilnius observatory was used.

The Tsar’s army soldiers helped Tenner to perform observations, built the points and temporary astronomic observatories and guarded them during the most of geodetic and astronomic activities.
It should be noted that Struve’s observations were very accurate. This was proved by the unbelievable accuracy of angular and linear observations in the 19th century. Error of angle measurement was 0.57” in Livonia, 0.87” in Finland, where the conditions for observation were the most complicated, and 0.68” in the Lithuanian arc. Accuracy of arc measurement was 1/232390 (4 mm per km).

Fig 4. The triangulation network established by Tenner in 1816–1829, Struve arc (in red) triangulation chain crossing Lithuania

The accuracy of the Struve’s arc triangulation chain was strictly checked after its connection to the Prussian triangulation network in 1832–1834. Triangulation networks of different countries were connected thanks to the efforts by F. W. Bessel, after measuring almost 200 km length triangulation chain between Trunco and Klaipeda, which ended triangulation network sides established by Tenner. After comparison of the common side length, computed
from baselines measured in Palanga and Königsberg, differences were below 0.15 m, and relative errors were lower than 1/200 000.

In the 1841, F. W. Bessel has computed and published Earth ellipsoid parameters based on the observation results of the connected triangulation networks. These have been used in many European countries.

By using Struve geodetic arc observation data, the meridian arc of 1° was computed for each of the twelve parts of the Struve arc: 1° length of the Northern part was 111,589 km; of the Southern part – 111,230 km; of the two Lithuanian parts 111,373 km and 111,292 km respectively.

**Scientific and practical importance of the Struve Arc**

The Struve geodetic arc has a significant scientific role. It was a longest arc measured at that time, running from Northern shore of Norway to the Black Sea. The Struve's arc was 450 km longer than arc of William Lambton and George Everest measured in India almost at the same time (1800–1843). The Struve's arc was the first one crossing territories of a few countries. Activities of the arc establishment demonstrated excellent collaboration abilities of scientists from different countries in solving problems related to research of the Earth form. 100 years have passed since establishment of the Struve arc until similar multinational measurements took place in East Africa.

The Struve arc was the most accurate arc of that time. Once more the exceptional accuracy of the geodetic measurements, achieved in the 19th century should be stressed. The best available geodetic and astronomic instruments, some of them especially produced for this purpose, were used for measurements. During the observation process the geodetic measurements methodology were elaborated. New method suggested by Struve was applied for measurements of angles, and method by Bessel for determination of the astronomic latitude. The astronomic longitude of MESKONYS point was determined extraordinary by using burned gunpowder.

Struve's arc being distinguished for its length and observation accuracy, invented new possibilities for the improvement of the Earth ellipsoid form and size determination. Bessel was first to use the data of the not yet completed Struve's arc for determination of the new ellipsoid parameters. Results achieved by Struve and his colleagues were used in all the subsequent computations of the Earth ellipsoid parameters, based on triangulation. They were
used in the course of a century, until methods of satellite geodesy for the determination of the ellipsoid parameters were elaborated.

The improved data on the form of the Earth ellipsoid were important not only in geodesy, but also in astronomy, navigation, cartography. Because of this new data, navigation in the seas was improved, distances to the celestial bodies were re-determined, and better cartographic projections were developed for the accurate mapping.

The arc was important for determination of the Earth form and dimensions, for improving of the geodetic observation methods, for developing practical precise geodetic activities. In the course of establishment of the Struve geodetic arc, solution of scientific problems was harmonised with practical tasks. Along with expanding triangulation networks, up to the 20th century the Struve arc chain was used as geodetic reference for mapping of territories that arc was going through. The arc chain of triangulation had a similar application in Lithuania too.

The active participation of astronomers from the Vilnius University in observations of the Lithuanian part of the arc and the successful results improved their competence in the field of geodetic astronomy and enlarged the instrumentation of the University astronomic observatory. Geodetic activities resulted in establishment of the Department of Geodesy at Vilnius University in 1826. The first head of the department was M.A. Antanas Sakinis (Antoni Szahin). The department existed until the University was closed in 1831.

The Struve arc measurements are important as achievement of science, as well as in its cultural aspect. Close cooperation in working out scientific, political, cultural and financial problems was going on among different scientists of many countries: geodesists, astronomers, and producers of instruments.

The Struve geodetic arc is one of the greatest scientific and technological achievements of the 19th century. This object manifests unique international connection of science and culture.

**Commemoration of the Struve Geodetic Arc**
The suggested method for commemoration of the heritage object required that every involved country locate at least two Struve’s geodetic arc points, and arranged everything according to the national requirements of heritage institutions. All countries involved collaborated in locating, investigating and commemorating sites of historical measurements since 1994. These sites are located using available geodetic observation data by means of the recent measurement methods as well as satellite geodesy.

The Struve geodetic arc, as one of the world’s foremost scientific and technical achievements is presented for the international society with 34 selected points. There are from one to six points in the following countries: Norway – 4, Sweden – 4, Finland – 6, Russia – 2, Estonia – 3, Latvia – 2, Lithuania – 3, Belarus – 5, Moldova – 1, Ukraine – 4. Other arc points are patronized at national level.

Every country has selected its most famous sites. The National Land Service at the Ministry of Agriculture of Republic of Lithuania and the Institute of Geodesy of Vilnius Gediminas Technical University have suggested commemoration of the three Struve’s arc geodetic points, namely GIREISIAI, MESKONYS and PALIEPIUKAI, which already belong to the National Geodetic Network. The suggestion was approved by the Commission of National Heritage of Republic of Lithuania and the Department of Culture Heritage Protection of the Ministry of Culture of the Republic of Lithuania.

Nomination was proposed to the UNESCO’s World Heritage Committee in January 2004. World Heritage Committee has included the Struve geodetic arc in the World Heritage List in July 2005. World Heritage Committee approved that the Struve geodetic arc meets even three criteria for inclusion into the World Heritage List.

The Struve Geodetic Arc was presented to the public at the Lithuanian Meskonys geodetic point in June 16th, 2006.

References


From Struve Arc to Tartu Hotel

Lina Färje
Architect, Stockholm, Sweden
a99_lfe@arch.kth.se

Starting my diploma work at the architectural department at the Royal Institute of Technology in Stockholm I found this really strange and interesting about a new World Heritage that caught my attention. Not knowing what this would lead to I decided to take the Struve Arc as a starting point for my work.

Tourism and travelling is an old but still increasing phenomenon changing people’s perspective and their perception of the surrounding environment. Something architects work with daily. The creation and appreciation of tourist sights is strongly enhanced by the increasing number of World Heritage sites. These often work as a catalyst for the development of the tourism industry and the maintenance of an environment that in other way might have lost its function.

So where does the Struve Arc fit in? It doesn’t really have anything to visit, has it? And where can you learn about it? It is too big and too small at the same time. Nevertheless this seemed like the ideal monument, something that can’t disappear, the measurements are not even physically there, and has this historic importance that makes anyone fascinated. At least this is my experience telling about this to people who haven’t heard of it before.

I got interested in these places, the actual measuring points. What can you experience there? What is their quality? I visited Tartu just because it was the easiest one to reach and to find, and found out that this was the charm with the other ones. You really have to search to find them! I love the description of one of the of the points as “internationally known as a
faraway place”. This is how they should remain. Tartu is a good point for my project. It can benefit from the attention and has the capacity to gather people without so much knowledge to become interested.

The measurements are of course a central part of the World Heritage. The triangulation as a technique, and the development until today with satellites. They are not there to experience today and were only used in the moment of measuring. The connection between the measuring points, how can they be experienced today? Is it necessary to be at the actual point or could it be somewhere else? How could these make sense in Tartu?

Representations of places are common in museums, dioramas and maps telling about different situations realistic or not so much. To put the Struve Arc into a museum would be to go against its principles. It should only exist camouflaged by another reality.

With the concept of a hotel it fell into place. The measurements became the hotel rooms, distinguishing one point from another, all individual, possible to visit and experience. The touristic aspect was obvious and also well needed in the place, the city of Tartu. It could be used by people interested in the Struve Arc, or not, perhaps becoming interested. Showing something you can’t experience at the other points, all gathered in one place, a virtual Struve world.
A short history of the obelisk at Fuglenes, the northernmost point on the Struve Geodetic Arc

Bjørn Geirr Harsson
Statens kartverk, Hønefoss, Norway
bjorn.geirr.harsson@statkart.no

In 1853 there was a meeting between Struve, Hansteen and Selander where they planned the finalization of the meridian arc project they had run. Struve told that he had got money for a monument at the southernmost station on the arc, and he asked Hansteen if he would be able to make something similar in Hammerfest, the northernmost point on the arc. We have a very good description on what happened when he came back to Norway and started the work to get a monument for the Hammerfest point.

Hansteen got the necessary money for an obelisk and used a famous architect to form it. In August the next year a nice obelisk was shown to the public in Christiania (called Oslo today) before it was shipped north to Hammerfest. In October 1854 documents shows that it is placed where the hut for astronomical observations had been before. It is interesting to see that on the obelisk itself the year 1852 is written (in Roman numerals) as the final year for the project, while 1855 has later become the official year for completion of what today is called the Struve Geodetic Arc.

In the archives it is mentioned that the obelisk was hidden away during the last years of World War 2, and was put back again in 1947. The archives, however, could not say anything about what happened to it. When I visited Hammerfest in March 2003 I happened to meet one of the persons who had taken care of the obelisk in the war time, and he gave the whole story on tape recorder how they had hidden the obelisk at a place near the hospital. The reason was their
fear for an allied attack on the Nazi submarine base situated near by the obelisk at Fuglenes.

When all Finnmark was burned by the Nazi in October 1944 and the population evacuated southwards the obelisk had been brought to Trondheim. First in 1947 it was brought back to Hammerfest, since not a single house existed in Hammerfest when the war ended in 1945.

At least two times later the local council in Hammerfest has tried to move the obelisk to another place since they wanted the ground used for new buildings. But The Norwegian Mapping Authority refused in both cases to accept any moving of the obelisk, so no moving took place. Since the Struve Geodetic Arc has been accepted on the UNESCO World Heritage List no more proposals of moving the obelisk are expected.

In the last years the Hammerfest municipality has shown great interest for the story about the Struve Geodetic Arc, and will in the next years spend a lot of money to make an even nicer place around the obelisk at Fuglenes. The Struve Geodetic Arc is so far the seventh Norwegian cultural object on the World Heritage List.
On Comparison of the three Meridian arcs in Lapland

Vitali B. Kaptyg

Department of Mathematical Geography and Cartography,
Russian Geographical Society, St-Petersburg
vitalikaptjug@yahoo.co.uk

In his extensive historical narration of Swedish polar travels and research ("Ulligh Latitudes", 1993) Dr. Gösta H. Liljequist included a brief note on the 2nd and 3rd arc measurements in Lapland, and remarked: "no studies of differences between these [of Selander – Struve] measurements and those of Maupertuis and Svanberg seem to have been made" (p. 334).

Why so? Were Svanberg’s and Selander’s measurements of secondary importance for science? Nothing of the kind! In 1844 Struve talked in Stockholm about the linkage of the “Russian arc” triangles with those of Svanberg, and hoped that the Swedish Academy would contribute to extending the latter measurement as far northwards as possible. Such a hope looks unlikely if the measurement was not accurate enough. The 1845 final decision by the Academy was, however, to carry out a new (the 3rd by order) arc measurement in Lapland.

Unfortunately, no direct comparison, hence, no studies either could have been made between Struve’s and Svanberg’s arc measurements on the same cause as previously it had been the case between Svanberg’s and Maupertuis’s works. Although the comparison was a "hot" scientific question of his time, Svanberg “was unable to find any of the signals used by Maupertuis”, except for two (Liljequist, ibid.), for obscure reasons – may be added, considering that several of the 11 stations of 1736 had been marked in rock. On the other side, in 1845 Wrede and Selander made, apparently, no or not successful search for the markers of Svanberg, thus providing Struve with no material to compare. In his “Arc du méridien
..."(1857; 1860; 1861) Struve was only able to compare one triangle side – the only identical one in Svanberg’s and Woldstedt’s (a Finnish colleague of Struve) triangles. This direct comparison looks rather intriguing: the side Torneå Landskyrkan – Kemi kyrkan between the two church buildings was found:

9178.960 toises in length, after Svanberg (1805, p. 100), and
9179.566 toises in length, under the azimuth of 96° 10' 06.4", after Struve (1861, II, p. 51),
with a disagreement of circa 1: 15000 for the distance of 18 km only.

Of course, this uncertainty could have been arranged using modern techniques but it would not yet speak for the whole picture. From scientific and cultural points of view importance of further direct comparisons between the works by outstanding geometers is obvious; hence, the need for launching a campaign of search for rock and stone crosses left by Maupertuis, Svanberg and Selander. After all, where the rock tops or stones have got to, and, really, should the marks be searched for to meet surveyors' interest only?

Rather general mathematical comparison between the 1st, 2nd and 3rd arc measurements in Lapland yet can be made. The known contributions of the kind only reached resulting lengths of 1° of latitude to compare; this method cannot be considered correct enough in numerical terms, let alone clear display of the factors involved – an important characteristic just at present when the task is allotted to make the World Heritage Struve Geodetic Arc more popular.

Following is the known basic equation for exact computation of the length of an arc-of-meridian segment on an arbitrary ellipsoid; it is accurate within 2 ppm at the latitudes and space of northern Scandinavia, which is more than sufficient:

\[ S = a \left( \frac{AB''}{\rho''} \right) \left( 1 - e^2 \right) \left\{ (1 - e^2 \sin^2 B_m) \uparrow -3/2 \right\} \]

where: \(a\) – semi-major axis, \(e^2\) – squared eccentricity, \(AB''\) – amplitude of a segment of arc of meridian, in arcsec, \(\rho'' = 206265\), \(B_m\) – mean latitude of the arc segment, the arrow means involution. As to appropriate ellipsoids, several ones are available; that of A.R. Bonsdorff (1888) will be applied, it provides with the best approximation of the Struve arc curvature of its own shape, with \(a = 3272563\) toises and \(e^2 = 0.00666867\). Now one should extract initial values of the quantities measured by the workers (astronomical latitudes \(\varphi\) will be taken for ellipsoidal latitudes \(\beta\):
Maupertuis: $AB'' = 0^\circ 57' 28''.67, Bm = 66^\circ 19' 42'', S = 55023$ toises.

Svanberg: $AB'' = 1^\circ 37' 19''.57, Bm = 66^\circ 20' 10'', S = 92778$ toises.

"Selander": $AB'' = 2^\circ 5' 13''.83, Bm = 67^\circ 15' 21'', S = 163222$ toises (after Struve).

The latter line presents not strictly Selander’s results, but those computed with Lindhagen’s observations added; Struve took the value of the Tomna Landskyrkan latitude observed by Lindhagen because the value observed by Selander had not been submitted. However, that circumstance counts for nothing in this general comparison. Thus, calculation results in the following, so to say, “true” values $S^*$ of the lengths of the measured arcs in view:


Comparison of the values shows that:

Maupertuis “overvalued” his $S$ by 1: 260, or his $AB''$ should be bigger by approx. 12";

Svanberg “short-measured” his $S$ by 1: 2900, or his $AB''$ should be less by 2".0;

“Selander”’s $S$ was “short-measured” by 1: 2000, or the taken $AB''$ should be less by 5".1.

The two alternatives following from the calculation are, of course, only the extreme cases of the errors in question, and the truth is, as always, somewhere in between those, provided there is no big external systematic influence. However, taking into account the specific instruments used by the workers for measuring the angles and baselines, one can hardly admit such great errors in the meridian distances neither for the Selander measurements, nor even for those of Maupertuis. On the contrary, well-known errors due to various sources of influence in astronomically observed amplitudes or latitudes seem much more believable for each of the three arc measurements; most probably, they are responsible for the greater part of the discrepancies just shown. Previously the "astronomical" errors have often been stated responsible for the too big value of the meridian arc of Maupertuis.

As to Svanberg’s result, it looks the best among the three ones although rather enigmatic after Struve’s collation; the measurement of Selander, generally, seems not very accurate. Was that really so?

To answer this question one should compare the “Selander-Skogman-Agardh” arc segment (Tomna Landskyrkan Stuor-Oivi) with the two adjacent ones.
1. The segment in the northern Finland (of “Woldstedt – Lindhagen – Struve”) has one common point Torneå Landskyrkan with the Lapland one, its latitude was observed by Lindhagen. Woldstedt observed the latitude at the other end (station Kilpinäki). The initial values for the evaluation of the segment are:

\[ AB'' = 3° 11’ 39’’.32, \quad Bm = 64° 13’ 55’’, \quad S = 182 794 \text{ toises.} \quad (\text{Struve 1861}) \]

Computation comes to \( S^* = 182 710 \) toises, in other words, “the Finland team” either “overvalued” the \( S \) by 1: 2 200, or the amplitude \( AB'' \) should be bigger by 5’’.3. The discord is contrary and nearly equal to that in the Lapland arc segment thus giving a reason to suspect a shift in the value of the astronomical latitude of Torneå Landskyrkan.

2. The segment in the Norwegian Finnmarken (of “Klouman – Lindhagen – Struve”) has one common point Stuar-Oivi with the Lapland one, its latitude was observed by Selander and Agardh; although they resulted in a disagreement of 1 arcsec, for the further consideration that amounts to little (Selander proposed the mean, and Struve agreed to that). Lindhagen observed the latitude at the opposite station Fuglenaes. The initial values:

\[ AB'' = 1° 59’ 12’’.83, \quad Bm = 69° 40’ 35’’, \quad S = 113 754 \text{ toises.} \quad (\text{Struve 1861}) \]

Computation comes to \( S^* = 113 728 \) toises, in other words, “the Norwegian team” either “overvalued” the \( S \) by 1: 4 400, or the amplitude \( AB'' \) should be bigger by 1’’.6.

Basing on Struve’s extended research into accuracies of the astronomical and geodetic work within the Finland and Finnmarken segments one should reject the supposition of serious observation errors of the two results just considered. Struve estimated the internal accuracy of Lindhagen and Woldstedt’s latitudes with the probable error \( \pm 0’’.06 \) - \( 0’’.08 \), i.e. with the maximal possible error within \( \pm 0’’.3 \). The observations of Selander and Agardh, by no means less accurate, reveal that the overall error could really extend up to \( \pm 0’’.5 \) or even a bit more. On the other side, Struve attributed to the value of the Finnmarken arc length the probable error \( \pm 1.8 \) toises, i.e. the maximal possible error \( \pm 5.4 \) toises or c. 50 ppm. The longer Lapland segment was considered, due to the better universal instrument of Selander, more accurate: the probable error \( \pm 1.7 \) toises, i.e. the maximal possible error \( \pm 5 \) toises or c. 30 ppm. The
northern Finland segment, yet longer, was proved by Struve the best accurate among the three (9 ppm maximal error).

A simple scheme drawn on paper can finally help to arrange the discrepancies within the three Struve Arc segments. By having them attributed mostly to the observed latitudes (on the above-mentioned reasons), one has no choice but admitting *appreciable external factors influencing the four astronomical latitudes, particularly much the latitude of Torneå Landskyrkan*. Another vote for this conclusion can be found in Struve’s “Arc du méridien ...”, where the great computer man revealed nearly *fourfold discord* of the counter-azimuths of the closing geodesic of Lapland relative to the three other northern sections (v. II, §118). Of course, those “factors” are well-known deviations of the plumb line due to unavoidable geomorphological influence. The analysis like this had to be fully explicated in the third volume of the Struve work, unfortunately, not accomplished due to an incurable illness of the worker.

In many respects Selander’s work could be compared with that by Struve on the Baltic arc segment: similar social status of the geometers, impossibility to leave their other official duties, hence, long duration of the field work, similar size of the measured space, the way of marking the stations and so on, even such peculiar detail as assistance by lieutenants of the marine. It is a matter of great regret that the first trans-Lapland measurements of the heights and arc-of-meridian by Selander have never been published. The most of the *theoretical* disagreement within his general result is amendable; as to real accuracy of his geodetic work, direct comparisons would only “dot the i’s”. At least, the both Selander terminals luckily remain safe and allow comparisons; his other rock marks, except for that at Perra-vaara (Perävaara) have yet to be found, as well as, of course, his invaluable archive.

Coming at last to Svanberg’s result, one can notice that it has suspiciously *similar minus in the geodetic length* as in the unfavourable Struve’s comparison. Generally, real quality of his geodetic work could be quantified by disclosing, how much the geodetic amplitude $AB''$ between the island of Mälaren and Paltavaara differs from the value $1^\circ 37' 17''.6$ (by $2''$ less than Svanberg’s amplitude $A \varphi''$) or, by introducing the known values of the plumb line deviation into Svanberg’s latitudes. Direct re-measurements go without saying. Yet the Svanberg measurement, contrary to Selander’s one, has been successfully accomplished and published by the worker, thus providing European science with the only arc of meridian in “high latitudes” for at least half a century, up to 1857.
INDEX

of field and other important documents
related to the Scandinavian segment of the World Heritage Monument
Struve Geodetic Arc

Part 1: Russian archives, version of June 8, 2006

Vitali B. Kaptlig

Department of Mathematical Geography and Cartography,
Russian Geographical Society, St-Petersburg
vitalikaptlig@yahoo.co.uk

Introduction: On the History of the SGA Manuscripts

To perform a meridian arc measurement through the vast East-European plain had been a persistent idea of men of science both in Europe and in Russia since at least 1721 (the first measurements by J.-N. De l’Isle in 1737). However, the next century provided better opportunities for “measurements of degrees” in Russia. They really started in 1816 thanks to efforts by two great national workers in geodesy and surveying: military surveyor Carl Tenner and astronomer Wilhelmin Struve. Struve’s authority at the court of Russian Tsar Nicholas I helped to win a considerable extension of the first successful operations, both southwards through modern Ukraine and Moldova (headed by Tenner), and — due to international cooperation and support by Oscar, King of Sweden and Norway — north of the Gulf of Bothnia to near North Cape. The entire enterprise took about 40 years, no wonder it produced a huge volume of documents. Those were in Russian, German (Struve was a native German), French (the language of Russian high life), Swedish and Norwegian, both originals and copies, of specific and of general nature. The basic field material was published in Struve’s and Tenner’s final descriptions, which cover the entire 25-degree measurement in a possibly
comprehensive way, with poorer content only regarding the works in Lapland performed by Swedish surveyors under Prof. Sclander's guidance. For yet unknown reasons, the Stockholm Academy of Sciences has never realized the 1853 coordinated suggestion of a separate publication describing the meridian arc operations through Lapland and Finnmarken as far as the Arctic ocean.

Original Struve Geodetic Arc (SGA) manuscript documents were never kept at one place, due to both inter-institutional and international nature of the arc measurements in view. The largest volume was preserved in the archive of former Pulkovo (original spelling: Pulkowa) Observatory near St.-Petersburg, where W. Struve was director from 1839 to 1862. With the exception of his later years of life the outstanding astronomer was in permanent charge of scientific guidance and publication of final results of the unprecedented geodetic enterprise. Field registers, diaries, records, reports, calculation sheets and related correspondence were all kept in full order in Pulkovo and had been in permanent demand by Struve and his associates until the end of the publication process (three editions within 1856-1861). On the other hand, a large volume of the documents related to the "Russian Arc" was kept in the archive of the Russian Military Topographers' Corps (MTC). Those collections were relative to the arc segment measured by military surveyors under Teucer's guidance, it stretched from the river Daugava southwards to the mouth of the Danube. Smaller collections of SGA documents, mostly correspondence, were kept at the Empire University of Dorpat, at the Russian Empire Academy of Sciences, at the Russian Ministry of Public Instruction and at the Russian Geographical Society. Because only a few reports of the SGA workers have been published by Struve, the vast correspondence, wherever it may be, still hides many important, although unknown details of the SGA measurements.

At present the surviving volume of the "Pulkowa" SGA manuscripts which relate to the Scandinavian part of the SGA is kept in the St.-Petersburg Branch of the Archive of the Russian Academy of Sciences. Here one can see field material, calculations, metrological research records, drawings and maps, letters and reports by the workers, draft copies of published material, official correspondence related to the works under Struve's responsibility, incl. those on the Scandinavian arc segment. Other Russian archives, except for the latter item, do not have important documents relating to that arc part.

Outside Russia, a substantial volume of important field material and related correspondence highlighting the SGA extension across Northern Scandinavia originally had to be kept in the archives of Stockholm Royal Academy of Sciences, the National Land Survey of Sweden and
Norwegian Mapping Agency, as well as in the personal archives of participants of those events: Selanders, Wrede, Skogman, Lilienhoek, Agardh, Hansteen, Klemman, Lundh, particularly – Lindhagen, etc. The author would welcome suggestions of co-operation in search, recognizing and indexing surviving materials.

Content and Structure of the Index

The Index presents the following types of archive material related to the SGA:

1) registers of field and special indoor measurements and research, either original or copied by the observers themselves; those may relate to trigonometrical (horizontal or vertical angles), base (linear) or astronomical measurements – all these are marked with the code letters T, R and Δ correspondingly;

2) notebooks, cards or just sheets of paper containing mathematical calculations, often inseparable from investigations related to the computed item – these are all marked with the code CI;

3) diagrams, instrument situation plans, maps and other drawn material – these are marked with the code D;

4) manuscript (text) material such as reports, letters etc., which contain important or just interesting details or circumstances which luckily could have been recognized within the mostly illegible German or French handwriting – these are marked with the code M.

Thus, every type of material included in the Index is marked with a particular letter code, just to make them convenient for fast recognition. Further, the material is separated into Finnmarken (Norway) and Lapland (Sweden and Finland) are segments distinguished by Struve himself. Spelling of geographic names corresponds to that used in the sources involved.

Explanation on some details.

a) Before the Index a summary list is given of the related folders and their respective general volumes expressed in paper sheets (including notebooks, if any). The folders, fortunately, are all kept in the same archive just mentioned. The address for making requests is:

Mrs. Irina V. Tumina, Doctor habil., Director,
The St.-Petersburg Branch of the Archive of the Russian Academy of Sciences,
1, Universitetskaya nab. 199034 St.-Petersburg, Russia.
b) Years of "field works" correspond to original documents; they indicate time spent on the reconnaissance survey and the measurements themselves.

c) In the 2nd column of the Index notations within square brackets, e.g. [1854], mean that this information (e.g. year "1854") was specified after another reference source rather than after this same document.

d) In the 3rd column of the Index each archive reference is presented as numerals separated by dots and commas; the numerals identify successively:

- the particular number of the archive collection ("fond" in Russian),
- the current inventory number within the collection ("opis" in Russian),
- the current number of the folder ("delyo" in Russian) within the indicated inventory, followed by:
- particular numbers of the sheets (not pages) presenting the document which may be given in the following forms:
  *) not specified, if the entire folder's content relates to the item described,

*) 3 : 5 means "from sheet 3 to sheet 5",

  *) 3 : means "beginning from sheet 3" (the end was not recognized),

  *) 5 : means "ending with sheet 5" (the beginning was not recognized),

*) 14r means the reverse side of the sheet No. 14.

In the future this version of the Index may only be added to items of drawing material and maps which are not yet available for inspection (see the related part of the Index). Other "geographical" parts of the Index, hopefully, will be published with time. The author will appreciate remarks or criticism on the form or content of the Index, so that the work would better meet requirements of research workers and lovers of SGA historical details.

Acknowledgements

My search for the SGA manuscripts began in St.-Petersburg as far back as 1997; it was a mere overture. A deeper identification of the found documents required more time; it became possible some later, thanks to co-operation with the International Institution for the History of
Surveying and Measurement (IIHSM) – a body within International Federation of Surveyors. Since 1994 the IIHSM has been aspiring to preserve the vast Struve Arc heritage, and the activities have successfully resulted in inscribing of a selection of the SGA field stations on the World Heritage List of UNESCO in July 2005. Care by the IIHSM for my archive work, financial support to catalogue still unavailable SGA documents was most important. The criteria of sorting out the documents as well as the final form of indexing them resulted from fruitful discussions with the IIHSM officers. Thus the merits of this publication largely belong to Messrs. J. De Graaf, IIHSM director; D.A. Wallis, treasurer and J.R. Smith, honorary secretary to the Institution; it is my pleasure to acknowledge their efforts and express my gratitude to them. I am also much grateful to Dr Nina Moskovchenko, presently research worker of the Museum of Pulkovo observatory, for her continuous contributions to my work. I am indebted to Dr habil. Irina Tunkina and other staff of the St.-Petersburg Branch of the Archive of the Russian Academy for our fruitful long-term co-operation. Thanks are also to Dr Alexey Litvin of the State Archive of the Russian Federation (Moscow) and Dr Natalia Goreceva of the State Military-historical Archive (Moscow) for providing me with "navigation" information within vast variety of possible sources. Special thanks are to Dr Maria Matveeva of the Archive of the Russian Geographical Society for important technical help and consultation at early stages of my archive experience. Importantly, recognizing of many documents became possible thanks to permanent reference to F.G.W. Struve’s “Arc du méridien...”, the 1861 edition in Russian, which has been in my unusually long possession in the Library of the Russian Geographical Society, and it is my pleasure to express cordial thanks to the Library manageress, Mrs. Svetlana Savina.
LIST OF THE ARCHIVE FOLDERS INVOLVED

2.1/1850.2 - [107], 703.7.155 - [446, 7 notebooks], 703.8, 703.12.67 - [88], 703.12.68 - [61],
703.12.339 - [21], 703.12.340 - [42], 721.1.32 - [440], 721.1.79 - [1530], 721.1.83 - [337],
8 notebooks], 721.1.84 - [739], 721.1.85 - [837], 721.1.86 - [412, 10 notebooks],
721.1.87 - [86, 3 notebooks], 721.1.88 - [61, 3 notebooks], 721.1.90 - [1142].

FINMARKEN ARC SEGMENT (Norway)
Field works: 1845, 1847, 1850 under the guidance of Hansteen.

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<th>CODE</th>
<th>DESCRIPTION</th>
<th>ARCHIVE REFERENCE</th>
</tr>
</thead>
</table>
| T & CI | Field registers ("Trigonometriske observationer...") for 1846-
| | 1847, 1850, copied by Kleuman, in Norwegian; calculations and |
| | notes in German added by Lindhagen, sometimes by Struve.
| | [1850-1856]. |
| B | Field register, measurements at 3 base extension points,
| | original by Lindhagen of 1850, in German. |
| B & CI | Field register, measurements at Alten, original by Lindhagen of
| | July 1850, in German. |
| "Basismessung bei Alten. Tagebuch von Fred. Klouman, 1850",
| clean copy, in German. [> 1852]. |
| A | Field registers of the latitude and azimuth observations at
| Fuglenæs, originals by Lindhagen of 1850, in German. |
| CI | Register by Lindhagen of the investigation of the Struve base
| apparatus in Pulkowa in May 1850, original, in German. |
| | Register by Lindhagen of the investigation of the same base
| apparatus in Bosekop, June to July 1850, original, in German. |
| | Registers [by Lindhagen] of final instrument investigations:
| "Zwei supplement Uebr zu den Operationen in Finmarken und
| Lapland", originals, in German. 1852. |
| | Computation material by Lindhagen, Struve, etc. related to the
| base measurement and astronomical observations, [1850-1856]. |
| D | Drawing material, maps |
| | Position of the astronomical instruments at Fuglenæs, scheme,
| ink, by Lindhagen [1850]. |
| | The monument at Fuglenæs, sketch, gouache, pencil, 18 x 30
| cm, signed in ink: "Christiania, 14/8 - 54, von Hanno", 1854. |
| M | Report by Lindhagen on results of the negotiations in Christiania
| and Stockholm, original of Nov.30, 1849, in German. |

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<th>CODE</th>
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| 721.1.83, 54 +161 (four
| successive notebooks 2 to |
| 5); 721.1.88, notebooks: |
| "X", "XI", "XII letztes". |
| 721.1.86, notebook "III",
| part 3a. |
| 721.1.86, notebooks "II",
| "III". |
| 721.1.86, 374 - 409 (the
| last notebook). |
| 721.1.86, six successive
| notebooks "IV" to "IX". |
| 721.1.86, notebook "I". |
| 721.1.86, notebooks "II",
| "III". |
| 721.1.87, notebooks "I",
| "II". |
| within 721.1.84. |
| 703.12.339, 703.12.340,
| 703.8 (not yet available). |
| 721.1.86, notebook "V",
| 200r. |
| 721.1.79, sheet No.1. |
| 721.1.85, 240 - 250. |
| Receipt by Klouman listing the Pulkowa instruments taken for measurements in Finnmarken, original of May 20, 1850, in Germ. | 721.1.85, 267. |
| Notes by Struve on the comparison of the Norwegian and Swedish baselines, his draft inscription for the proposed monument at Fuglaena, original letter, Sept. 1853. | 721.1.85, 612 - 617. |
| Report by Klouman on general circumstances of the field works: "Momenter til Indledningen ved Beskrivelsen over Gradmaalingen i Finnmarken", original in Norwegian; translation to German added by Lindhagen, dated March 20, 1856. | 703.12.67, 102 : 112. |
| Notes on the course of the measurements works: "Suplement til...Struves historiske Beratning om Gradmaalingen...", original by "K." [Klouman], in Norwegian. [> 1851]. | 721.1.85, 582 : 583r. |
| Description of the monument at Fuglaena, translated by Lindhagen from an unavailable original by Klouman: "Über das Gradmessungsmonument bei Fuglaena", in German. [< 1857]. | 703.12.68, 11 - 12, the inscription: 14 - 14r. |
| Letters by Struve (draft copies), Haasteen and Klouman (originals), in German. 1848-1855. | within 721.1.85. |
**LAPLAND ARC SEGMENT** (Sweden and Finland)
Field works: 1845–1852 under the guidance of Wrede (1845) and Selander (from 1846).

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<tr>
<th>CODE</th>
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<tbody>
<tr>
<td>T</td>
<td>Field registers by Lindhagen of the base measurement at Över-Torneå, original, in German, 1851.</td>
<td>703.7.155, notebooks “I”, “II”.</td>
</tr>
<tr>
<td>A &amp; D</td>
<td>Field registers by Lindhagen and Wagner of the astronomical observations at Torneå, original, in German, incl. a situation plan, June to September 1851.</td>
<td>703.7.155, notebooks “III” to “VII”.</td>
</tr>
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<td>C1</td>
<td>Results of linkage of the Swedish and Finnish stations, [Lindhagen], original, 1851.</td>
<td>721.1.84, 548.</td>
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<td></td>
<td>Final angular directions observed from Torneå to Bälljätavan, incl. base extension stations, copy by Struve from an unavailable original by Selander, in German, September 1853.</td>
<td>21/1850.2, 81 × 82.</td>
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<td>Cards of computation of the arc chain from Torneå to Alten, originals by Struve, in German, 1853.</td>
<td>721.1.90, 78 × 551 × 614 ×.</td>
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<td></td>
<td>Computation material by Lindhagen, Struve, etc. related to the base measurement and astronomical observations, [1851–1856].</td>
<td>within 721.1.84.</td>
</tr>
<tr>
<td></td>
<td>Situation plan to the astronomical observations near Torneå, [by Lindhagen, 1851].</td>
<td>721.1.84, 537.</td>
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<td></td>
<td>Diagrams of the Över-Torneå base extension, [by Struve ?], originals, August 1853.</td>
<td>721.1.90, 516, 519 ÷ 520.</td>
</tr>
<tr>
<td>M</td>
<td>Some notes on the accomplished works, in 2 original Selander’s letters of 1845–1846, in German.</td>
<td>721.1.85, 54 ÷ 56r.</td>
</tr>
<tr>
<td></td>
<td>Report on the arc measurement works accomplished by 1851, [original ?], in Swedish, [Selander, or Lindhagen’s copy].</td>
<td>721.1.85, 334 ÷ 335.</td>
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<tr>
<td></td>
<td>Notes on results of the comparison of the Swedish and Norwegian baselines, in Struve’s original letter, in German, September 1853.</td>
<td>721.1.85, 612 ÷ 617.</td>
</tr>
<tr>
<td></td>
<td>Letters by Struve (draft copies), Selander, Wrede, Lindhagen (originals), in German. 1844–1855</td>
<td>within 721.1.85.</td>
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</table>

* Note. Two important Selander’s contributions reported to Struve on July 27, 1853 and on August 10, 1855 contained some of the final results relating to the Swedish arc segment; until they were received Struve had been able to perform only interim linkage computations south and north of the segment, basing on the data submitted by Lindhagen. However, Selander’s original reports have not been found.
Two Centuries of University of Tartu Observatory

Reet Mägi
Director of the University of Tartu History Museum

The University of Tartu Observatory has served science and education for almost two centuries. The building was completed in 1810 and soon became an important science centre. Today the Observatory is internationally highly regarded as a UNESCO World Heritage site. Unfortunately, the building is in bad condition and the restoration plans have yet to attract the necessary funding. The following presentation will give an overview of the history of the Observatory as an architectural monument, spanning the time that has elapsed from its conception to the present, as well as the plans for the building's future as a museum.

The University of Tartu was founded in 1632 as a Swedish university. It is the oldest and the only universites-type educational establishment in Estonia. The University was suspended in 1710 due to the Great Northern War and reopened in 1802 as a Russian Imperial University, however, being intellectually a German institution for most of the 19th century. In 1919 it was re-established as the Estonian national university. At the beginning of the Second World War, Estonia was occupied by the Soviet Union and the University remade in the image of the new rule. In the course of the war the Soviet and the German occupations succeeded one another several times, the Soviets finally prevailing as winners of the war. The Republic of Estonia only regained its independence in 1991.

The Observatory was completed in 1810 - eight years after the University was reopened. It was built in the centre of the city on the Dome Hill, in the vicinity of which many other university buildings were being erected including the University's Main Building, the Anatomical Theatre, the Library, and the Clinic. The site the Observatory was built on holds a special meaning in the history of the city of Tartu. All previous buildings at this site used to be symbolic of the city in their time. From the middle of the 1st millennium the cite was
occupied by a stronghold of ancient Estonians which played an important role in the development of the future town and city. The first mention of this stronghold, in a Russian chronicle, occurred at the same time as Tartu was first referred to in historical records. In the Middle Ages the Bishop’s Castle and thus the centre of the Tartu Bishopric was situated in the very same place. When it was built, the Observatory became a symbol of Tartu as a university city.

From plans to construction

The Observatory’s current site was actually not the builders’ first choice. Initial plans saw the Observatory being built in the tower ruins of the medieval Dome Cathedral, also on the Dome Hill. In the conception of University architect J. W. Krause the ruins were to be fashioned into a ‘spiritual temple’ for the University with the Library in the choir, the Observatory on the western towers and the University Church in the nave between the two. The University astronomers, however, did not agree with Krause’s idea of the ‘temple’ and it was eventually decided to build the Observatory on the site of the former bishop’s castle.

The first draft plans for the Observatory were sketched already in 1803. The shape of the building was set out in 1804 by professors Knorre, Phaff, and Parrot. The observatories of Gotha, Göttingen and Uppsala served as examples. In 1807 Krause elaborated the sketch into a building plan that became the basis for the actual construction. The Observatory was a simple building with two high halls, one in the East and the other in the West wing. The domed tower was positioned above the entrance. Construction work started in 1808 and the building was completed in 1810.

F. G. W. Struve became Director of the Observatory in 1818. It was on his initiative that the Tartu Observatory received a Fraunhofer Refractor, the world’s largest telescope of the time, in 1824. The observation tower proved to be too small for this instrument. A new rotating tower with an open gallery was therefore built in 1825 after the design of G. Fr. Parrot. The tower suffered some damage during the Second World War. The most extensive modification of the building was carried out in 1952, when the Observatory’s West Hall was converted into a two-storey office wing in order to win workspace and the staircase in the centre of the building was rebuilt. However, a large part of the Observatory has been preserved in its original condition to the present day, amongst others the tower’s rotation mechanism and the observation skylight cover system of the East Hall.
The complex of the Observatory also includes the Director’s residence and an annex built in 1819-1821 as well as three smaller observation facilities. The largest of these is the so-called Petzval’s tower, which was erected in 1897.

**The Observatory as a Museum**

Tartu Observatory eventually became too cramped and its location in the centre of Tartu no longer suitable for observations. A new observatory complex was set up in the village of Tõravere near Tartu. In 1964, astronomers moved out of the old Observatory to continue their work in the new facilities. In fact, the Old Observatory on the Dome Hill had already been considered too small some time before. Yet, various plans to extend the complex had failed to be carried out. Thus, in Struve’s time a second residence was intended to be built for Observatory employees. The 1909 and 1910 plans to build a new observatory were dashed by the onset of World War I. A 1945 plan intended to replace Struve’s former residence with a building containing laboratories, a lecture hall, and employees’ apartments.

In 1971 an Astronomy Museum was opened in the Observatory, which was affiliated to Tartu Town Museum. Those interested in astronomy, students in particular, had had access to the Observatory before, though. In the 1930s there were two to three thousand such visitors a year. After World War II, the Observatory’s astronomers continued to host an average of 150 excursions a year with 3000 visitors.

In 1996 the Observatory was restituted to the University and the Town Museum’s branch closed. The Ahhaa Science Centre founded by the University started to use some of the rooms in the Observatory and its annexes. The Centre was founded with the aim to popularise science, and has proved a very successful venture. It runs at least one large exhibition a year at various leased locations. In addition to those, smaller events take place at the Observatory all year round and the building itself is open to visitors.

By 2008, the Ahhaa Science Centre will move into a building specifically designed for it in the centre of Tartu. This means that the Centre’s administration will move out of the Observatory’s annexes that it occupies currently. Still, the Centre’s cooperation in the Observatory’s museum part with the University History Museum will continue. There is also an astronomy club that is active in the Observatory and is mentored by professional astronomers. The club organises frequent observations that can be attended by members of the public.
In future Tartu’s oldest observatory will function as a part of the University of Tartu History Museum. The plans for restoration and interior design of the building have been completed. Tartu Municipal Council and the University’s leadership support restoration and adaptation of the building to function as a museum. The problem is that presently available funds are insufficient to fulfil all their aspirations. The University and the City have started collaboration to prepare a large project that will comprise the restoration of the Observatory and the opening of the museum as well as reconditioning the entire surrounding park. Unfortunately the realisation of the project in its present form has been delayed. At the moment the most realistic way of accomplishing it seems to take things forward step by step. With this in mind, the University is currently preparing to apply for funding from prospective Norwegian and European donors.

During its history, the Observatory has been a top-level science centre, one that is destined in the future to become a museum. This means that the building with its dignified history will persist as an educational institution – hopefully capable of continuing its scientific pursuits by helping win young people’s hearts over to science.
The Arc of the 30th Meridian through Eastern Africa

J R Smith
Hon. Sec. International Institution for the History of Surveying & Measurement
jim@smith1780.freeserve.co.uk

Abstract

If one looks at a map of European arcs of meridian and parallel at the beginning of the 20th century there will be seen to be a plethora of such arcs. Turning to Africa there was the complete opposite. Other than the arcs of Eratosthenes (c 300 BC) and the short arcs by LaCaille (1752) and Maclear (1841-48) the continent was empty. It was in 1879 that David Gill had the idea for a Cape to Cairo meridian arc but 1954 before it was completed.

This presentation concentrates on a description of the 30th Meridian Arc in East Africa. The only other comparable arc in the British Colonies at the time was that through the centre of India by Lambton and Everest between 1800 and 1843. The usefulness of such arcs was highlighted in 2005 with the inscription by UNESCO of the Struve Geodetic Arc on the World Heritage Monument list. A practical extension to the Struve Arc Monument is the 30th Meridian Arc since there is a connection between the two.

Lindsay Braun, [1], detailed the political machinations of the history of the 30th Arc; here it is hoped to fill in other aspects of this work including some facts and figures. After all, surveying is a profession of number gathering and crunching.

The Indian Arc took 43 years to complete but the African Arc took 75 years. The 125th anniversary of the inception of the 30th Arc and the 50th anniversary of its completion, were both celebrated in 2005. Plaques were unveiled in 2004 and 2005 respectively at the northern
and southern terminals. Such long-term projects illustrate the dedication of the participants in striving towards goals, the fruits of which, they possibly would not live to see.

Reference


N.B. This paper is due to be presented at an International Cartographic Association symposium in Utrecht in August 2006.

J.R. Smith
24 Woodbury Ave,
Petersfield,
Hants GU32 2EE,
UK
e-mail jim@smith1780.rrceserve.co.uk
F. W. Bessel and Geodesy

Tõnu Viik

Tartu Observatory, Tartumaa 61602, ESTONIA
Tartu University, Ulikooli str. 18, Tartu 50090, ESTONIA

1. Introduction

Renowned mathematician, astronomer and geodesist Friedrich Wilhelm Bessel started his geodetical research when it was already clear that Isaac Newton had been right in theorizing that the Earth was oblate. This had been ascertained by the geodetical measurements of meridian arc between Torneå (Tornio) and Kittisvaara by Pierre Louis Moreau de Maupertuis and Alexis Claude Clairaut between Kittisvaara and Torneå in 1736-1737 [1], remeasured by Jöns Svanberg in 1801-1803 [2], and by Charles-Marie La Condamine and Pierre Bouguer in 1735-1743 between Tarqui and Cotchesqui in Peru [3]. At the same time all the attempts to express the figure of the Earth as a perfect ellipsoid failed. This called for an explanation that the curvature of the meridians along the latitude was not constant, consequently the Earth was not a perfect ellipsoid. By and by the scientists were convinced that the figure of the Earth cannot be represented by simple mathematical formulae at all.

Still the scientists wanted to specify the parameters of the ellipsoidal approximation and also to learn how much the figure of the Earth differed from that of an ellipsoid.

This goal was to be accomplished by meridian arc measurements and by determining the gravitational force at different points of the Earth. At that time the first goal could be achieved only by triangulation and for the other goal the scientists measured the gravitational force as a function of latitude. Though Bessel dedicated a lot of his energy and time to both of these goals, only the first of them will be considered in this paper. All the Bessel's papers on
geodesy were collected in the third volume of the book edited by R. Engelmann [4]. The majority of these papers was previously published in Astronomische Nachrichten [5-12].

2. Theory of geodetic measurements in Bessel's work

Bessel's first study in geodesy was connected with the Danish arc measurement by Heinrich Christian Schumacher in Holstein which Schumacher directed from 1817. These studies were in the form of letters to Schumacher who later published them in his journal Astronomische Nachrichten. Bessel studied the problem of substituting the triangles on an ellipsoid by triangles on a sphere. If this was possible in the framework of given accuracy then one could easily substitute the spherical triangle by a planar one taking into account that all angles of that triangle should be decreased by one third of the spherical excess [5].

The transition from measuring triangles on the ellipsoid's surface to measuring spherical triangles is complicated also by the fact that on an ellipsoid the triangles are formed by geodesics, i.e. by the shortest possible lines between two points on that surface. But the measurements by using levels give us the angles between the vertical sections of the Earth. Bessel was the first to indicate that these triangles do not coincide and elaborated a method how to convert one to another. It appeared that this conversion was possible only for triangles with sides less than 150 km if one wanted to maintain a passable accuracy.

In 1826 Bessel solved one of the basic problems in geodesy – how to find the latitude and the difference in longitudes of a point B by measuring the latitude at point A and the angle between the meridian and the direction to point B [6]. Gauss had solved this problem by using his theory of surfaces but this solution was accurate enough only for rather small distances. Bessel gave his classical solution in trigonometric series and this solution was very general. Widely used, too, since Bessel compiled also large tables which were usable for any ellipsoidal model of the Earth.

Bessel gave also the definition of geoid, though the term itself was coined by a German mathematician Johann Benedict Listing only in 1873. In a paper published in 1837 Bessel wrote: "Es ist ein wesentlicher Unterschied zwischen der physischen und der geometrischen Figur der Erde... Denkt man sich also die Erde mit einem Netz von Kanälen überzogen, welche mit den Meeren in Verbindung sind und durch dieses gefüllt werden, so fällt die Oberfläche des ruhigen Wassers in denselben mit der geometrischen Oberfläche der Erde zusammen" [8]. Nowadays the surface of a geoid is described exactly in this way.
Bessel studied also the so-called general reductional problem in geodesy. He investigated the influence of the difference between the geoid and the ellipsoid on the results of the astronemo-geodetical measurements. He showed that however small the differences one should take these into account. He proposed to consider as basic points not the points themselves but the points of their normal projection on the surface of ellipsoid.

3. Geodetic measurements in practice

Bessel's most important contribution to practical geodesy was the arc measurement in Prussia. The motive for this measurement was a plan to join the Russian and the West-European
triangulation chains at their closest points near Königsberg where Bessel worked as the university Professor of astronomy and mathematics. It was in 1829 when the Russian government wrote a letter to the Prussian government in which they expressed a wish that director of Königsberg University Observatory Professor Bessel should join these two chains. The triangulation in the southern parts of Russia was then ongoing and lead by major-general Carl Tenner. Bessel was quick to understand that this task offered a great opportunity to determine a more accurate figure of the Earth.

In order to carry out this task Bessel had to build a chain of triangulation signals to south-western side of the Prussian chain at Trunz–Wildenhof and in north-eastern side of Memel (Klaipeda–Lepaizi which was part of the Russian chain (Fig. 1). The arc to be measured joined Trunz (now Milejewo in Poland) with the lighthouse tower in Memel, altogether 196 kms. The chain of triangles was built at the shore of gulf Frisches Haff, then through Königsberg and Semland peninsula and at last through Kurisches Haff [13].

That Bessel chose as a baseline for these measurements a distance of mere 935 toises or 1823 meters (1 toise = 1.849 m [16]) shows unequivocally that he was a man not troubled by traditions. Up to this time the mantra of geodesists had been that the longer the baseline the better. A baseline of 10 kms was considered normal though to measure it with greatest possible accuracy presented serious problems. According to Bessel: “Wir zweifelten nicht, durch diese Hilfsmittel eine kürzere Linie mit grosser Genauigkeit bis zu einer beträchtlich längeren vervielfältigen zu können und sahen daher eine grosse Länge der zu messenden Linie für weniger erheblich an, als ihre Nähe bei den Prüfungsapparate der Messstangen und als die Forderung, ihre Länge durch ein ungeformtes und vielfältige bedingungen zu seiner Prüfung darbietendes Dreieckssystem auf die Dreiecksseite Galligarten-Condehnen übertreten zu können.” According to Bessel’s better judgement for obtaining a certain accuracy by measuring long baseline one may reach the same accuracy by means of measuring a short one if only we measure both the angles and distances extremely carefully with best possible instruments – with “Hilfsmittel”!

The baseline was measured between the manors Trenk and Medniken in five days in August 1834. It began on a small hill, then sank a bit, went over peat-like surface on pastures and grasslands. After having covered 200 toises the baseline reached the fields and more grasslands till it ascended towards the endpoint at Medniken. In the evening of August 11th they reached the point A only 226 toises away – evidently the team was not experienced enough! Next day, having measured the base line from A to Medniken they returned to Trenk and repeated the measurement. Since Bessel was not satisfied with the results, the distance
between Trunk to A was measured once more. After painstaking calculations Bessel obtained the result – the base line reduced to the sea level was 934.993124 toises.

Bessel was also very careful in fixing the baseline points. In deep holes (5 feet, 1 Prussian foot = 31.385 cm) with stony foundation the granite boulders were established on top of which the brass cylinders were fixed into the stones. The crosses on these cylinders marked the respective endpoints. Above the ground the points were marked by brickwork pillars, on which a sandstone cube was placed. Again a brass cylinder was placed on that cube while the cylinder extended half an inch from the cube. The cylinder was axially perforated.
Bessel measured the lengths of the baselines with an instrument of his own construction. The instrument consisted of four rods, each approximately 2 toises long, 12 lignes wide and 3 lignes thick (1 ligne = 2.256 mm [18]). The rods were made of two bars – one from iron and the other from zinc, with half the width of that from iron, put one on another and rigidly fixed at the endpoint. The two-metal combination allowed to take the thermal deformation into account without measuring the temperature. The rods were placed in wooden boxes in order to prevent the direct influence of solar rays and humidity on them. The rods were positioned on the baseline with a clearance between them while each rod was supported at seven points on rollers. The clearances were left there deliberately since this effectively excluded accidental displacements of the rods. The length of a clearance was measured with a glass wedge which could be moved out from the rod by means of a screw (the so-called Bessel wedges). The rods were placed on special thick oak platforms in direction of the baseline, and the accuracy of the placement of the rods was checked by levels and passage instruments, i.e. astronomically.

Bessel paid also exclusive attention to scrupulously comparing of the length of his instrument with the standard Toise de Pérou in Paris. It should be mentioned that in 1839 a new standard of length of three Prussian feet was introduced in Prussia which was elaborated by Bessel and which rested upon his theory and experiments with pendulum [14]. During the process of comparison Bessel placed the rods in the water in order to obtain a homogeneous distribution of temperature in them and in that way to reliably estimate the influence of temperature on the rods.

This Bessel instrument was so successful that it was used by geodesists up to the beginning of the 20th century. It should be mentioned that in connection with this work Bessel solved the problem of the least sagging of a rod supported at two points [15].

As far as the arc to be measured was at an angle of 40 degrees of the meridian for some points of the arc both the latitude and longitude had to be measured. Bessel avoided the longitude measurements by measuring the respective azimuths because the longitude measurements were not sufficiently accurate at that time.

For angle measurements Bessel used the transit which was made by Ertel in Munich and designed by Schumacher. The diameter of the horizontal limb was 38 cm and that of the vertical limb – 19 cm. One could read the azimuth with accuracy of 2 arc seconds.

Among Bessel's principal helpers were major Johann Jacob Bueyer from the Prussian General Staff (later General) and lieutenant Kuhlenkamp – perhaps a relative of the owner of a commercial firm in Bremen were Bessel started his independent life.
For sighting the faraway objects Bessel used a polished copper hemisphere with an approximate diameter of 21 cm, covered with silver and placed on the cylinder protruding from the sandstone cube. During a sunny day a reflected light from that hemisphere could be seen at a distance of 10 km when sighted through a 15 inch theodolite. When it was cloudy a white 2 by 2 feet square with black line of thickness of 10 inches on it was used for sighting. For longer distances – 40 to 50 km – a Gaussian heliotrope was used.

The results of geodetic measurements were to be checked by astronomical observations in order to determine the latitudes and longitudes at some points. This problem was solved by a transportable passage instrument. Here Bessel used his own method which ruled out the instrumental errors for determining the latitudes with this instrument. This method was also used by General C. Tener in Russia.

The real work for joining the geodetic networks in Prussia and Russia was to begin in 1831. But Bessel and Bayer could barely start their reconnaissance when epidemic of cholera broke out in Prussia, one of the worst in its history. Bessel and Bayer could start their work again only in 1832.

After the detailed check of the results the two chains were joined and the results were published in a book [13].

4. The Bessel ellipsoid

One of the Bessel's greatest geodetical results was defining a new mathematical model of the Earth's figure. Nowadays we know that as the Bessel ellipsoid. In defining the figure he used his own arc measurement which covered a mere 1° 30' 28".980. In addition to this he made use of 9 more arc measurements, namely the Peruan (3° 7' 3".455), the first East-Indian (1° 34' 56".428), the second East-Indian (15° 15' 40".728), French (12° 22' 12".74), English (2° 50' 23".497), Hannover (2° 0' 57".42), Danish (1° 31' 53".306), Russian (8° 2' 28".907) and Swedish (1° 37' 19".565). When summing up all the measurements, an arc of almost 50 degrees was covered. The number of astronomical points was 38. Bessel saw his task in finding such an ellipsoid where the distances from its surface to real surface of the Earth were as small as possible. At the same time he planned to orient the axes of the ellipsoid in such a way that all the arc measurements were best presented.

In 1837 Bessel published a paper where he brought out the results for the elements of the Earth's figure: the polar and equatorial radii, the oblateness, analytical values for one
degree of meridian and parallel as functions of latitude [9]. He gave also the curvature of meridian and the distances from the Earth's center (perhaps it is interesting to point out that the author of his papers in Astronomische Nachrichten was not simply Bessel but at first “Herr Professor und Ritter Bessel” and later “Herr Geh. Rath und Ritter Bessel”!)

Unfortunately it happened that four years after the publication of this paper the Paris Academy of Sciences reported of a substantial error in their calculations – they have used a wrong “Berechnungsart” in elaborating the distance between Montjouy and Mola on Formentera island which resulted in an error of approximately 70 toises (136.5 m). Louis Puissant brought this error before the French Academy and the Academy responded in taking care of repeating the calculations by four independent persons: Largeteau, Daussy, Mathieu and Puissant, and their results coincided in the interval of 5-6 toises. However, Bessel still checked all the French calculations himself and only after he had found that his results coincided very well with the averaged French results, he made all his calculations anew and published the results in 1841 [12]. For the value for the oblateness he obtained 1/299.153.

The Bessel ellipsoid was in use close to 100 years in different countries all over the world. Even the final processing of Struve arc measurements was done on the Bessel ellipsoid [17]. Only after a hundred years enough new data was obtained and the figure of the Earth was more accurately specified and nowadays the accepted result is 1/298.257 ([16], International Earth Rotation and Reference Systems Service, 1989) , so Bessel's error was only 0.3% while Struve's error was 1.2%!

5. Conclusion
Besides of being a brilliant astronomer and mathematician, Bessel was also a very good geodesist. The principles of organising the observations and the theory of both personal and instrumental errors that he had elaborated were all used not only in astronomical but also in geodetic observations. In his works the mathematical apparatus of astronomy and geodesy attained a powerful development. Still we may consider the determination of the exact figure of the Earth one of the Bessel most important results.

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Figure 2 & 3. The Bessel Base line instrument.
D.G. Lindhagen and the arc measurements

Hans-Fredrik Wennström
Lantmätiverket - National Land Survey of Sweden, S-801 82 Gävle
hans-fredrik.wennstrom@lm.se

D.G. Lindhagen was a key person in the final phase of the work on the Struve geodetic arc. He took part in the measurement of sections 11 and 12, called "Lapland" (Laponia) and "Finmark" (Finmark) stretching from Tornä to Hammerfest (at 65° 49' 48" and 70° 40' 12''). The two sections cover 4° 51', or about one fifth of the total length of the arc.

Daniel Georg Lindhagen, or Daniel Lindhagen, was born on July 27, 1817 in the province of Östergötland, in Askeby parish east of the city of Linköping. His father Carl Israel Lindhagen was the assistant vicer (kommunister) in the parish. Daniel was sent to Uppsala for university studies and in 1837 passed his first exam, permitting further academic studies. In 1843 he was given an assistant position (emeritus) at the observatory—still the old Celsius building from 1742 in the centre of the city, and not very well furnished with instruments. This lasted for about three years, till 1846, when he got his BA degree (fil.kand.). (Uppsala got a new observatory in 1853.)

As the university observatory was very modest, travels to European observatories was a necessity. In the autumn of 1846 Lindhagen was a member of a small party travelling to observatories in Germany, the Netherlands, France and Italy. As the youngest in the party he had to do the bookkeeping and take notes. Lindhagen’s journey ended in Pulkovo in 1847, the new Wilhelm Struve observatory, opened in 1839, and with the most modern equipment. Gustaf Svanberg (professor of astronomy at Uppsala) had provided a letter of introduction. Within short Lindhagen got a provisional position (1847). He made observations and could in 1848 defend his thesis in Uppsala for the Ph.D. degree (Collinder, p. 47). At the end of 1849
he was given an astronomer's position at Pulkovo. Uppsala university gave him the status of docent next year (1850), with the permission to stay in Pulkovo.

Lindhagen and the Struve arc

In the mid 1840's almost all sections of the arc measurement were completed, but the full length demanded a northern extension från Torneälven up to the Arctic Sea. Sections 9 and 10 through Finland ended in Torneälven. Swedish and Norwegian survey parties headed by Selander and Hansteen were organized. In the work that followed Lindhagen was participating both in Norway and Sweden to secure the quality and homogeneity of baseline measurements and astronomical observations.

In 1847 Lindhagen went to Christiania (now Oslo) to prepare for the Finnmarken section. In 1849 he was in north Norway and made the base measurement in Alten together with the Norwegian geodesist Klouman. In Hammerfest he made astronomical observations. In 1850 and 1851 he observed angles necessary to connect the northernmost points in the Finnish section with that of "Lapland" (in Torneälven, the Torné valley). Together with Selander he measured the baseline at Övertorneå (Ylitornio). The sections 11 and 12 made the arc complete after more than 35 years, which is marked on the obelisks at the termini, 1816-1852, and further to 1855 as on the title page of Arc du méridien. Lindhagen made all the calculations for (the northern part of) the arc. He was offered a good appointment by the Russian Academy of Sciences, but preferred to return to Stockholm (Collinder, p. 47).

In Pulkovo Struve lived with his family. Among the children was Olga (1830-1894), and Georg Lindhagen married Olga Struve in 1854. They soon moved to Stockholm.

In 1855 Lindhagen was assistant astronomer at the Stockholm observatory, then a part of the Royal Swedish Academy of Sciences. Lindhagen could resume astronomical work in Stockholm. A first publication was Om terrestre refractionens theori (1857). He also published a work on the comparison between the Struve double-toise and the copy made of it for the Swedish Academy of Sciences. It was communicated October 8, 1862 and published next year. In december 1862 he communicated the results of astronomical observations made by A.F. Nordenskiöld on Spitzbergen.

Popular astronomy

Lindhagen also began to work on a popular book on astronomy based on the Cours élémentaire d'astronomie from 1853 of C.E. Delaunay (born 1816, d. 1872). Lindhagen's
version of the book was called *Astronomiens grunder* ['Foundations of astronomy'] and in three volumes published in 1858, 1860, 1861. It was part of a series of popular science books (*Bibliothek i populär naturkännighet*, IV). Astronomy was later on covered in a similar way for the general public in works by e.g. Knut Lundmark (Lund observatory) and Östen Bergstrand (Uppsala observatory). Lindhagen also held a public lecture on astronomy at a society for the dissemination of knowledge in Stockholm.

Lindhagen was elected a member of the Royal Swedish Academy of Sciences in 1859. In 1860 he took part in an English solar-eclipse expedition to Spain. The date for the eclipse was July 18 (Lindhagen, 1860).

**Lindhagen's lecture 1862**

A small group of manuscripts shows Georg Lindhagen's interest in making astronomy and science more popular. Two of them are (unpublished) lectures at the Royal Swedish Academy of Sciences, March 31st, 1862 and 1863. The day is the yearly celebration of the date of the Academy's first statutes of 1739, and the audience usually included the King, nobilities and other guests. Lectures were not too a technical level.

With his thorough knowledge of arc measurements Lindhagen's lecture on March 31, 1862 was on the "The quest for the true Size and Shape of the Earth" (Lindhagen 1862 ms). He gives a summary of the measurements by Maupertuis and Svanberg and continues with the "combined russo-scandinavian arc measurement" and mentions the recent publication of the work (i.e. 1860). The arc is the greatest hitherto, and still Nature sets no limit for its continuation to Turkey, the Greek archipelago and Candia (Crete), by which an arc of 37° can be achieved. He also comments on the result of the section measured in Lapland. The length of a degree is 57188 toise from this section separately, which is almost identical to Svanberg's 57196 toise of 1802.

Lindhagen continues his lecture with the triangulation in Great Britain by colonel James and captain Clarke, and comments on the question of local attraction and includes also some information on the reduction in Great Britain of latitudes with regard to the nearby mountains. Next follows a similar description of the work in India, and the astonishing results from Kaliana, and the theory of Airy. The lecture was an almost complete briefing to the special audience regarding high precision geodesy of 1862.

The real purpose of the lecture might, however, to have been to make a plea for an arc measurement on Spitzbergen. Last year, i.e. 1861, a Swedish expedition had returned from Spitzbergen. It was headed by Otto Torell. The first suggestion of an arc measurement on
Spitzbergen came from Edward Sabine, who through his extensive journeys on the Atlantic to make pendulum observations also visited Spitzbergen. On the Swedish expedition of 1861 two young participants were detached to find suitable peaks for a chain of triangles that could be used for an arc measurement. The astronomer and spectroscopist Nils Dunér (1839-1914) from Lund had to work in Storfjorden, but this failed due to the heavy ice. Karl Chydenius (1833-1864), physicist from Finland, made his reconnaissance between Ross Island, 80° 48' N and a peak on 79° N (an arc of 1° 48'). Including Storfjorden it would be possible to measure an arc of 4° (Chydenius, 1862).

Lindhagen gives the details of the reconnaissance survey and concludes with the hope that an arc measurement on Spitzbergen could be a reality and not only just an idea. (A new Swedish expedition set off for Spitzbergen in 1864 but the degree measurement was postponed.)

In the Academy of Sciences Lindhagen delivered a lecture also in 1864. It was on "Light" and focused on the most recent news on the speed of light— the measurements by Fizeau and Foucault in Paris. At this time the speed of light was mainly of astronomical interest. A few more manuscripts but for other audiences covers astronomy and the method of least squares.

**Mitteleuropäischer Gradmessung**

From 1862 Lindhagen was involved in a new arc measurement. Shortly after the publication of Struve's work, a new long arc measurement was discussed in Berlin, by J.J. Baeyer. At first German states Prussia and Saxony coordinated geodetic ambitions as their representatives met in Berlin in 1864 and formed the Central European Arc measurement (**Mitteleuropäischer Gradmessung**) with an arc running from Berlin southwards and northwards. Already in 1862 and 1864 Lindhagen made observations in a triangulation chain on the West coast of Sweden.

At the meeting in Berlin 1867 the new organization **Europäische Gradmessung** was founded. Sweden and Norway were invited to take part. The Swedish participant in 1867 was Lindhagen, who then is one of the "founding fathers" of the international IAG.

An arc through central Europe could be calculated from existing triangulation chains if they had a high quality. In some cases such chains could be reobserved and provided with more baselines etc. Sweden already had a triangulation chain on the West coast, that could be upgraded to be included in the Central European Arc and serve as a link to Norway. Georg Lindhagen measured on the West coast of Sweden in 1862 and 1864.
In 1865 a Scandinavian collaboration took place that determined longitude differences between Stockholm, Copenhagen and Christiania (Oslo). Observers of longitude were Lindhagen in Stockholm, Schellerup in Denmark and Fearnley in Norway. The same year Lindhagen was given the name of professor, in line with this Danish and Norwegian co-writers on the longitude.

**Secretary of the Academy och Sciences**

In 1866 Lindhagen was elected permanent secretary of the Swedish Royal Academy of Sciences. A few more field seasons on the West coast (1866, 1867) seems to be the final geodetic and astronomic work of Lindhagen, who from now on was fully involved in the Academy of Sciences as its permanent secretary. Lindhagen stayed as secretary from 1866 to 1901, a period of 35 years.

**Spitzbergen arc measurement**

During the last years of the 19th century the plans for an arc measurement on Spitzbergen were revisited. A new generation of scientists were interested in a scientific expedition. An arc measurement could be included. A first reconnaissance tour in 1899 was sponsored by the Academy of Sciences. Funding was secured and the expedition took place in 1900-1901 in collaboration with Russian scientists. The arc was measured in its northern part by Sweden and its southern by Russia. It was published as a part of the multi-theme expedition. Among the participants were Edward Jäderin whose invar-wires for arc-measurement were used on Spitzbergen for the very first time (his earlier model was of steel). Also Tryggve Rubin was a participant on Spitzbergen, the man that in the 1920's measured a long section in Africa of the Arc of 30th Meridian, the project by Gill (cf Widmark, 2000).
Georg Lindhagen was a person taking part or giving advice in three arc measurements (Stuve, Central European/European and Spitzbergen). He was the Swedish delegate to the Europäische Gradmessung conference in 1867.

Today Georg Lindhagen is not so known (in Sweden) as his brother Albert Lindhagen (Clacs Albert L., 1823-1887), who drew a famous master plan of 1866 for the city of Stockholm with broad boulevards in the Haussmann style. Within the history of city planning his name is one of the foremost in Sweden. He also took part in the foundation of Stockholm's Högskola, the forerunner of Stockholm University.

Georg Lindhagen died on May 5, 1906. In 1909, when he should have been 90 years old, the Academy of Sciences issued a memorial medal (minnespeng) that in its brief motivation reads PER SEPTEM LUSTRA SECRETARIO, "for seven lustra secretary" (a lustrum is five years).

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Conference participants

Annika Hansson, Haparanda, Sweden
Karin Hollo, Tallinn, Estonia
Arunas Buga, Vilnius, Lithuania
Lina Färje, Stockholm, Sweden
Bjørn Geirr Ilarsson, Ísafjörður, Iceland
Vitalija Juozviciute, Vilnius, Lithuania
Vitalij B. Kaptnig, St. Petersburg, Russia
Reet Mägi, Tartu, Estonia
Anders Olsson, Gävle, Sweden
Jarmo Raita, Helsinki, Finland
Lars Snö, Haparanda, Sweden
Pekka Tätilä, Helsinki, Finland
Tõnu Viik, Tartu, Estonia
Hans-Fredrik Wennström, Gävle, Sweden

Special guests:
Kerstin Fredga, professor, Stockholm, Sweden
Inge Jonsso, professor, Stockholm, Sweden